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ABSTRACT

Two theories direct researchers in their efforts to find the optimum school or district size. The first theory holds that expenditures per student decrease as the size of the school increases. The second theory maintains that the first is true only to a certain enrollment level at which point the greater complexity of the school increases expenditures per student. Using samples of 100 elementary, 100 secondary, and 100 unit districts from Illinois, regression analysis showed the second theory to be more nearly correct. The optimum district size in terms of per student operating expenditures was 750 in an elementary district (K-8), 500 in a secondary district (9-12), and 5,000 in a unit district (K-12). (RA)

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OPTIMUM SIZE OF SCHOOL DISTRICTS RELATIVE TO SELECTED COSTS

A paper presented to the 1971 Annual Meeting
of the American Educational Research Association
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by

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PREVIOUS RESEARCH

One of the oldest approaches to obtaining efficiency in the public schools has been the quest for an "optimum" school size. Unfortunately this "optimum" has been almost as elusive to researchers as the Holy Grail was to King Arthur's knights. The problem, as Hickey (1969) has recently pointed out, is that we don't ask the right questions. At the outset we should determine whether this "optimum" size is being sought relative to (a) output, or to (b) costs, or to (c) educational services provided. We also need to determine just what units of analysis we are talking about, i.e., school districts versus individual high schools, or junior high schools, or elementary schools. The right question, it would seem, is to ask what the "optimum" size is relative to all three variables simultaneously, or at least for two of them, that is, cost and output. We hasten to inform the reader that we have not ask that question and hence must join a number of other studies in the purgatory of partial analysis.

No small part of this "optimum" size puzzle lies in determining the "true" shape of the function of size relative to outputs, costs, and services provided. If all these functions turned out to be linear it would be helpful since linear programing techniques could then be utilized to achieve a solution subject to several constraints. For educational services provided there would seem to be no great mystery. An abundance of research indicates that the function is, indeed, linear. That is, small schools provide less services and larger schools provide more services (McLure, 1964; Benson, 1965;

Thomas, 1968; Kiesling, 1968). We also know that the increased diversification of services in the larger schools results in higher costs (Bowser, 1969). Once that safe harbour is left behind, however, the sea gets much rougher and the linear assumption appears much more in doubt.

The size-output relationship is particularly perplexing. For California elementary schools in metropolitan areas Alkin and Benson (1968) could find no increase in mathematics and reading achievement associated with increased size after the socio-economic background of students and the expenditure per pupil had been allowed to operate. Since the size variable was not statistically significant the investigators did not try to determine the shape of the size-output function. Kiesling (1968) did explore the shape of the size-output function using high school data from the Project Talent survey. For several outputs the shape of the function is that of a parabolic arc with a positive linear component and a negative quadratic component. The "optimum" high school size relative to several achievement tests falls in the 1200 to 1600 ADA range. However, this U shaped curve is present only in the gross relationship between size and the output variables. When the socio-economic background of the students and the expenditure per pupil are allowed to operate the shape of the function changes and it then becomes linear and negative. That is, larger schools are associated with lower achievement test scores. Given these results we would have to concur with James and Levin (1970) that it is not very meaningful to talk about economies and diseconomies of scale with regard

to output; at least not until we have more information on the shape of the function.

Researching the cost-size relationship is an old activity for students of educational administration. A review of the secondary literature will show that there have been many such studies conducted (Cooper and Dawson, 1950; Stephen and Spies, 1967; James, 1969). A great many of these studies have reported that the cost-size relationship is not linear. Specifically they have reported that high per pupil costs are usually associated with both small schools and very large ones, with minimal costs for those in between. This is in keeping with economic theories of the firm where one expects to find both "economies and diseconomies of scale". That is, unit cost is usually higher for a small unit of output, but as the unit of output is increased unit cost per unit output decreases. However, as the unit of output is increased a point is reached where unit costs start to climb. Several reasons are advanced for this in the economic literature but they tend to boil down to (a) the indivisibility of some factors of production and (b) greater productivity resulting from a greater division of labor and specialization. Diseconomies are often associated with the costs of coordinating and managing the larger production processes.

As might be expected with so many cost-size studies being conducted, the research designs, units of analysis, and statistical sophistication of the investigators varies greatly. Even among the better studies there are problems. For example, some studies have simply assumed the existence of a

parabolic arc which is negative in the linear and positive in the quadratic (Hirsch, 1960; Riew, 1966). In these studies no attempt was made to statistically test the extent of departure from linearity. One study (Hanson, 1963) used a residual approach, that is, the residuals from a prior cost function in which the size variable had been deliberately excluded were used. The concept of economy of scale is fully supported in the Hanson study but the notion of diseconomies received less support. In three of the studies (Hirsch, 1960; Riew, 1966; Cohn, 1968) an attempt was made to control for quality of services provided. In the Riew study this was done by including such items as number of credit units offered, and the average number of courses taught per teacher within the general cost model. In Cohn's investigation this was achieved by such variables as average number of college semester hours per teaching assignment, and average number of different subject matter assignments per high school teacher being included in the cost model. Cost-size studies which attempt, no matter how crudely, to control for quality of services provided must be considered superior to those studies that do not control on this variable. This is the most important and serious reservation the researchers have about the findings reported in this paper. The cost-size relationship was explored without controls established for levels of services provided. However, unlike previous studies, the departure from linearity of the cost-size function was tested statistically rather than simply assumed to exist.

Two studies (Riew, 1966; Cohn, 1968) applied the differential calculus to the parabolic function to determine the minimum cost position. This resulted in very similar findings. Riew found optimum high school size relative to cost in Wisconsin to be 1675 ADA in the 1960-61 school year and Cohn found the optimum size of Iowa high schools to be 1500 ADA in the 1962-63 school year. Recalling the Kiesling optimum for achievement scores the 1500 ADA figure may well prove to be optimal on several criteria. The study reported here also uses the differential calculus to determine optimum size relative to cost.

THE BASIC QUESTIONS OF THE STUDY

Within the framework of this size-cost relationship four basic questions were asked:

1. At what size range are school districts in Illinois "too small" in terms of the "economy and diseconomy" concept?
2. Are there "large" school districts in Illinois that are in the "diseconomy" range?
3. What is the optimum size for school districts in Illinois, if indeed there is such an optimum size?
4. On the basis of the three types of school districts, will it be more economical to operate a unit district than to operate separate elementary and secondary school districts of comparable size to the unit district?

THE RESEARCH VARIABLES

The two basic variables used in the study were district size in terms of average daily attendance (ADA) and school expenditures. The school expenditures are the observed current expenditures per pupil in ADA in the district. In Illinois these expenditures are reported in what is called the "educational fund". Capital expenditure was not included.

Three forms of this cost variable were analyzed for their relationship with the district size.

1. The gross form. This is the actual observed expenditures per pupil without eliminating or holding constant any factor(s) that may influence cost in the district.
2. The residual form. This is the difference between the observed expenditures of the district and the expenditure level based on a linear and/or curvilinear relationship with the district's wealth in terms of assessed property valuation.
3. The administrative cost per pupil. This is a part of the current operating expenditures that pertains to administration.

These three forms of cost served as the dependent variables while the district size served as the independent variable. In the statistical treatment the variables were analyzed separately for the three types of school districts. This was a cross-sectional analysis and the individual school district was the basic unit upon which the data were collected.

THE SAMPLES

The samples for this study were drawn from public school districts in the State of Illinois, with the exclusion of the school district in the City of Chicago. For each type of district--elementary, secondary, unit--100 schools were selected to form the samples. 100 elementary districts

offering K-8 grades, 100 secondary districts offering 9-12 grades, and 100 unit districts offering K-12 grades. Since in this study a satisfactory representation of the variation of the district size is necessary, the proportional stratified sampling technique was employed. The 100 school districts for each type of sample have size ranges of: elementary districts from 49 to 9,733 pupils; secondary districts from 63 to 9,000, and unit districts from 111 to 32,000 pupils.

THE MATHEMATICAL ANALYSIS

The "economy and diseconomy" concept assumes that a curvilinear relationship exists between size and cost. High costs are usually associated with both small districts and very large ones, with minimal costs for those districts in-between. To determine whether this assumption holds the analysis utilized three tools:

1. The graphic method. This method was used only to visually determine whether the cost-size relationship does depart from linearity. This method was performed by classifying the school districts in the sample into size groups and computing the averages of these size groups. The averages were then plotted on ordinary graphing paper.

2. The statistical approach. This process was performed by fitting the "best fit" curve on the data. Mathematically, curvilinear relationships can be expressed by many equations. However, the theoretical construct upon which this study was based suggested the concave

parabola as a model of the cost-size relationship. This simple parabola is defined by the equation $Y = a - bX + cX^2$. This curve has two characteristics: (1) The curve is always symmetrical on both sides of the lowest point--the point where it stops going down and starts to turn up. The curve could then be cut into halves at the point of turning upward; one half would be the mirror-image of the other. (2) The curve has only one change or inflection point, i.e., from moving downward to moving upward. These characteristics make the simple parabola not very satisfactory to represent many types of relationships. However, it has some flexibility in that many different shaped curves can be represented by some particular arc segments of the parabola. (Ezekiel and Fox, 1959). Because of these characteristics other parabolic functions were also employed which involved the logarithmic transformations of some of the components of the equation of the simple parabola. Algebraically, the analysis was performed by polynomial regressions for each curve function up to the second degree order. (Draper and Smith, 1968). Statistically the parameters were estimated by the least squares method. The regression equations are as follows:

First degree order	Second degree order
1. $Y = a + bX$	$Y = a + bX + cX^2$
2. $Y = a + b\log X$	$Y = a + b\log X + c(\log X)^2$
3. $\log Y = \log a + b\log X$	$\log Y = \log a + b\log X + c(\log X)^2$
4. $\log Y = \log a + bX$	$\log Y = \log a + bX + cX^2$

where: Y = the dependent variable, cost

X = the independent variable size (ADA)

a = the constant term (intercept value)

b = the regression coefficient of the linear function

c = the regression coefficient of the quadratic function

The polynomial regression was employed so that both the linear and quadratic functions could be fitted to the data. The quadratic fit could then be compared to the linear fit to determine whether the quadratic function was a significant improvement over the linear function. The F-test employed to determine the improvement of fit is defined by the equation (Volk, 1958):

$$F_{(1/n-2)} = \frac{SS_{d2} - SS_{d1}}{SS_{a2} / n - 2}$$

where: SS_{d2} = the sum of squares due to regression of the second degree order

SS_{d1} = the sum of squares due to regression of the first degree order

SS_{a2} = the sum of squares about regression of the second degree order

n = the number of items in the sample

The shape of the curve will depend on the signs of the regression coefficients. If the value of the b coefficient is negative and the value of the c coefficient is positive the curve will be concave from above. If the value of b is positive and the c is negative the shape of the curve will be convex.

To determine which parabolic function best fits the data the indexes of correlation and determination were compared. The parabolic function that provided the highest indexes of correlation was used to describe the relationship between size and cost. An F-test was employed to determine whether the indexes of determination (R^2) are significantly different than zero. The test is defined by the equation (McNemar, 1969):

$$F = \frac{R^2 / m}{(1-R^2) / (N-m-1)}$$

where: R^2 = the index of determination

m = the number of parameters (regression coefficients)

N = the number of cases in the sample

3. The calculus application. In order to find the "optimum" size district relative to costs the first derivative of the function $Y = a + bX + cX^2$ was taken which is $b + 2cX$. Setting the first derivative equal to O and solving for X, the inflection point of the parabolic function will be determined by dividing the linear coefficient (b) by twice the value of the quadratic coefficient (c).

LIMITATIONS OF THE DESIGN

The study limited itself to the analysis of the relationship of district size and school expenditure, per se. The cost variable was not weighted nor analyzed for what it "buys" in terms of school programs the districts in the sample offer. Of the factors that could influence school expenditure in the district only one, the "wealth" of the district in terms of assessed valuation, was considered.

The quadratic function that was utilized in the analysis adequately described the relationship between size and cost, but a better relationship may be fully expressed by a more complex curve function.

RESULTS OF THE ALGEBRAIC ANALYSIS

The results of the least square regression show that of the four parabolic functions used in the analysis, the equation that appears to best fit the data for the three forms of cost variable is $Y = a - b\log X + c(\log X)^2$. The relationship of the cost-residual and size for the high school district, however, is best described by the equation $\log Y = \log a - b\log X + c(\log X)^2$. These equations provided the highest indexes of correlation and determination and were found to be significantly different than zero at the .01 level of significance (Table 1).

For all cost variables and for all types of districts the parabolic function was a better fit to the data than the linear function (Tables 2, 3, & 4).

Graphs of the functions are provided in Figures 1, 2, & 3.

PRINCIPAL FINDINGS

1. The "economy and diseconomy of scale" concept, as it applies to school operation was fully supported. It was evident that as the size of enrollment increased school expenditure decreased up to a certain point in the size continuum. When the enrollment exceeded this point per pupil costs start to climb.

2. The unit school district experiences economies of scale through a much greater segment of the size continuum than the elementary and secondary districts.

3. Size of the district in terms of pupil enrollment in ADA influence per pupil cost with or without holding constant the effects of the assessed valuation upon costs.

4. About 58 per cent of the variation in administrative cost per pupil is explained by the size of the unit district, while only 15 and 23 per cent are explained by size of the elementary and secondary school districts, respectively. It was also shown that the unit district experiences economies of scale on administrative costs through a greater segment of the size spectrum than the dual elementary and secondary districts.

5. The regression and calculus analysis of the three forms of cost variables with size have established the following minimum-optimum-maximum size values for economic efficiency:

a. Gross expenditure on size.

(1) Elementary district - minimum 250; optimum 750; and maximum 3,000 ADA.

(2) Secondary district - minimum 175; optimum 500; and maximum 2,000 ADA.

(3) Unit district - minimum 1,000; optimum 5,000; and maximum 35,000 ADA.

b. Cost-residual on size.

(1) Elementary district - minimum 125; optimum 500; and maximum 1,500 ADA.

(2) Secondary district - minimum 180; optimum 500; and maximum 2,000 ADA.

(3) Unit district - minimum 400; optimum 12,500; and maximum 50,000 ADA.

c. Administrative cost on size.

(1) Elementary district - minimum 400; optimum 7,500; and maximum 20,000 ADA.

(2) Secondary district - minimum 420; optimum 2,500; and maximum 12,000 ADA.

(3) Unit district - minimum 1,000; optimum 8,000; and maximum 40,000 ADA.

6. The contention that it will be more economical to operate a unit district than to operate elementary and secondary school districts of comparable size to the unit district was verified provided that the size of the unit district is at that level were the least-cost-combination of the unit district is less than those of the dual elementary and secondary districts combined. This enrollment level for the unit district in Illinois

was 1,500 ADA. As enrollment size increase from this level, estimates of per pupil cost for the unit district become consistently less when compared with estimates for the elementary and secondary districts of comparable size to the unit district. The difference becomes more pronounced the larger the unit district becomes until the optimum size is reached.

CONCLUSION

It can be concluded that on the basis of per pupil expenditures the unit districts are enjoying more of the benefits of economies of scale (least-cost-combination) than dual elementary and secondary districts. In Illinois there are quite a number of small and large elementary and secondary districts that can be considered to be operating in "diseconomy". This could be eliminated by reorganizing the dual districts into unified districts (K-12) of sufficient size to benefit from the least-cost-combination. Likewise, there are small unit districts that should be reorganized into larger units. For optimum efficiency the unit district should be organized with 5,000 ADA where feasible. A larger optimum size could be set at 12,500 ADA and a maximum of 20,000 ADA in areas where the population warrants. Each state department of education should conduct its own size studies. There is good reason to believe that the "optimum" size in one state is not necessarily the "optimum" size in another state. However, the methodology illustrated in this paper should be applicable to most studies which focus on questions of "optimum size".

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TABLE 1

F-VALUES FOR THE TEST OF SIGNIFICANCE OF THE INDEXES OF DETERMINATION[a]

	Index of Deter. (R^2)	F-Value
Gross Expenditure		
Equation: $Y_1 = a - b\log X + c(\log X)^2$		
Elementary District	.1764	10.39[b]
Secondary District	.1332	7.46[b]
Unit District	.1521	8.70[b]
Cost-Residual		
Equation: $d = a - b\log X + c(\log X)^2$		
Elementary District	.1369	7.70[b]
Unit District	.1296	7.22[b]
Equation: $\log d = a - b\log X + c(\log X)^2$		
Secondary District	.1444	8.19[b]
Administrative Cost		
Equation: $Y_2 = a - b\log X + c(\log X)^2$		
Elementary District	.2304	14.53[b]
Secondary District	.1522	8.71[b]
Unit District	.5844	68.27[b]

[a]To be significant with 2 degrees of freedom in the numerator and 97 degrees of freedom in the denominator $F_{.05} = 3.093$, $F_{.01} = 4.829$.

[b]Significant at the .01 level.

TABLE 2

THE REGRESSION AND CORRELATION COEFFICIENTS FOR THE RELATIONSHIP BETWEEN SCHOOL DISTRICT SIZE AND GROSS EXPENDITURE PER PUPIL

	Linear Corr. Coef. (r)	Index Corr. Coef. (R)	Index of Deter. (R^2)	F-value[a]	Std. Error of Est.	Constant Term (a)	Regres. Coef. (b)	Regres. Coef. (c)
Elem. Dist.	-.098	.42	.1764[b]	20.2[b]	71.34	2566.7	-1355.5	232.00
H. S. Dist.	.005	.36	.1322[b]	14.3[b]	82.06	2811.2	-1381.7	249.90
Unit Dist.	-.32	.39	.1521[b]	5.7[c]	41.23	1698.8	- 560.2	72.28

[a]For the F-test to determine whether there was an improvement of the quadratic function over the linear function

[b]Significant at the .01 level

[c]Significant at the .05 level

TABLE 3

THE REGRESSION AND CORRELATION COEFFICIENTS FOR THE RELATIONSHIP BETWEEN SCHOOL DISTRICT SIZE AND COST-RESIDUAL

	Linear Corr. Coef. (r)	Index Corr. Coef. (R)	Index of Deter. (R^2)	F-value	Std. Error of Est.	Constant Term (a)	Regres. Coef. (b)	Regres. Coef. (c)
Elem. Dist.	.25	.37	.1369[a]	8.78[a]	40.06	946.3	-521.5	100.2
H. S. Dist.[b]	.109	.38	.1444[a]	14.8[a]	58.9	2.8876	-1.7419	0.3185
Unit Dist.	-.30	.36	.1296[a]	4.24[c]	38.22	1083.1	-494.1	60.0

[a]Significant at the .01 level

[b]From the equation $\log d = a - b\log X + c(\log X)^2$

[c]Significant at the .05 level

TABLE 4

THE REGRESSION AND CORRELATION COEFFICIENTS FOR THE RELATIONSHIP BETWEEN SCHOOL DISTRICT SIZE AND ADMINISTRATIVE COST PER PUPIL

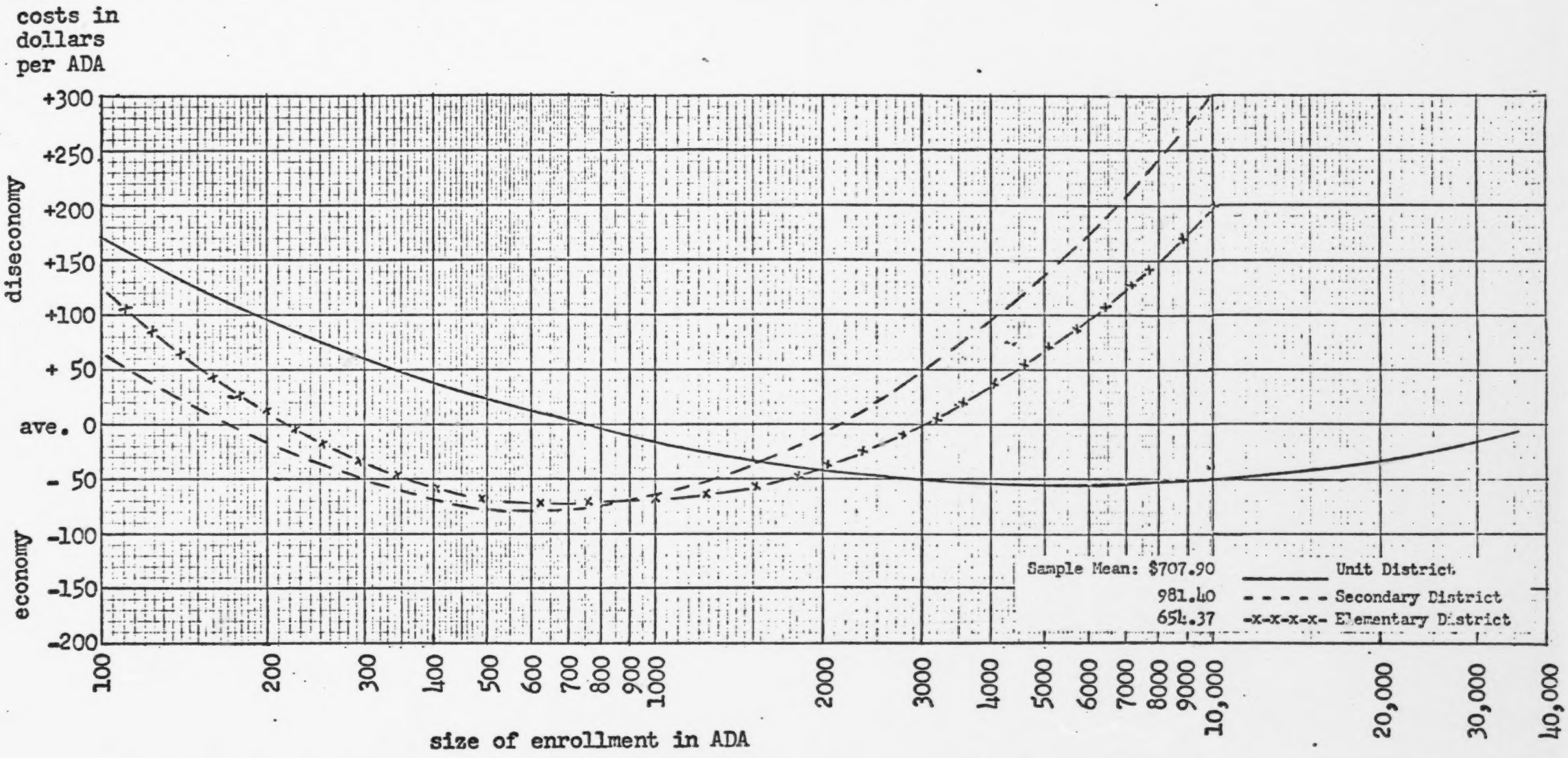
	Linear Corr. Coef. (r)	Index Corr. Coef. (R)	Index of Deter. (R^2)	F-value	Std. Error of Est.	Constant Term (a)	Regres. Coef. (b)	Regres. Coef. (c)
Elem. Dist.	-.42	.48	.2304[a]	3.97[b]	10.94	178.9	-76.74	9.99
H. S. Dist.	-.31	.39	.1522[a]	3.91[b]	35.90	693.7	-387.6	58.0
Unit Dist.	-.69	.76	.5844[a]	26.99[a]	13.40	305.17	-147.3	18.8

[a]Significant at the .01 level

[b]Significant at the .05 level

FIGURE 1

UNIT COST CURVES IN RELATION TO DISTRICT SIZE SHOWING DIFFERENCES FOR TYPES OF SCHOOL DISTRICT[a]



[a]Costs expressed in dollars above or below the mean of the sample.

FIGURE 2

COST-RESIDUAL CURVES IN RELATION TO DISTRICT SIZE SHOWING DIFFERENCES FOR THE TYPES OF SCHOOL DISTRICT

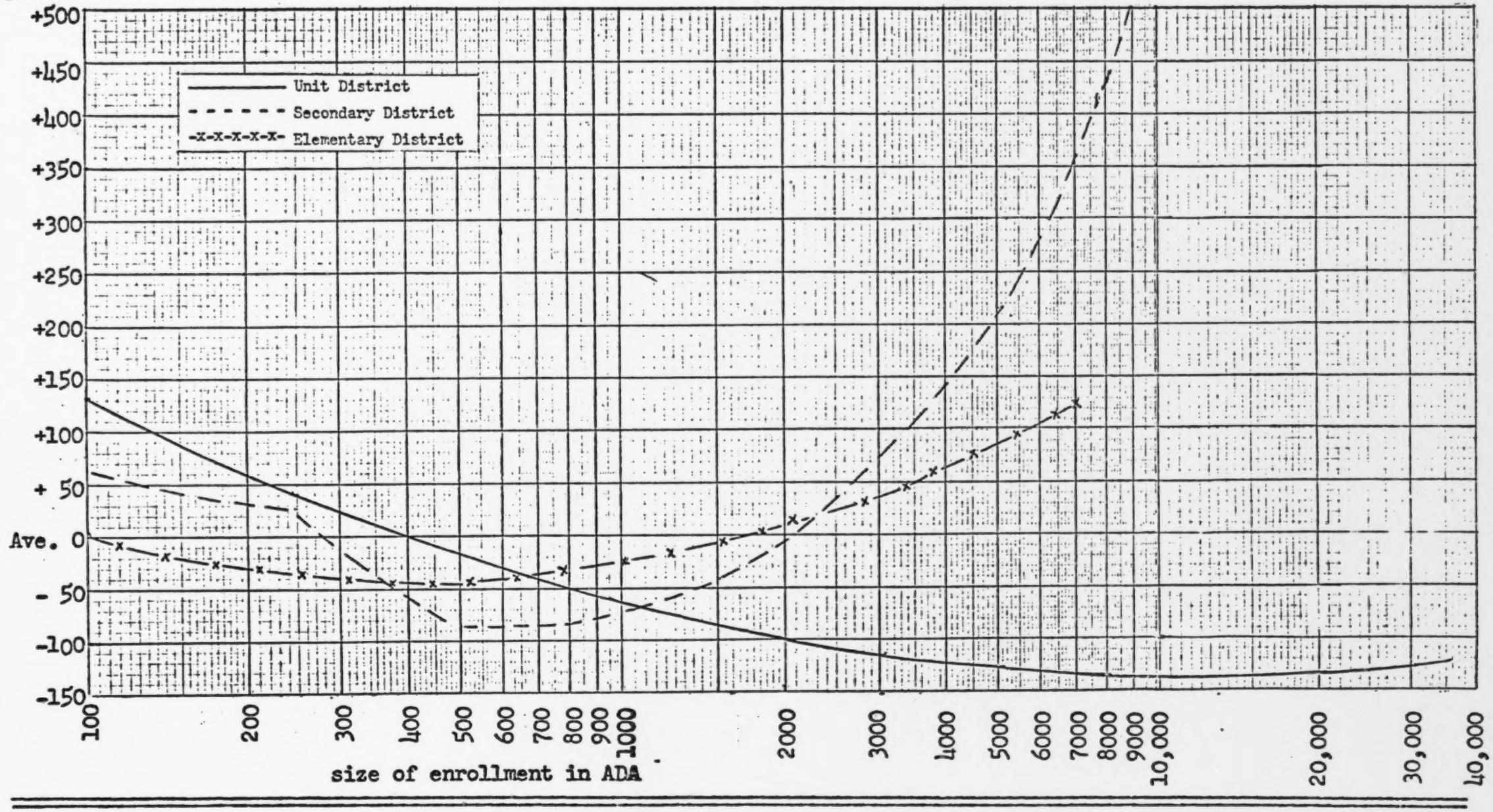
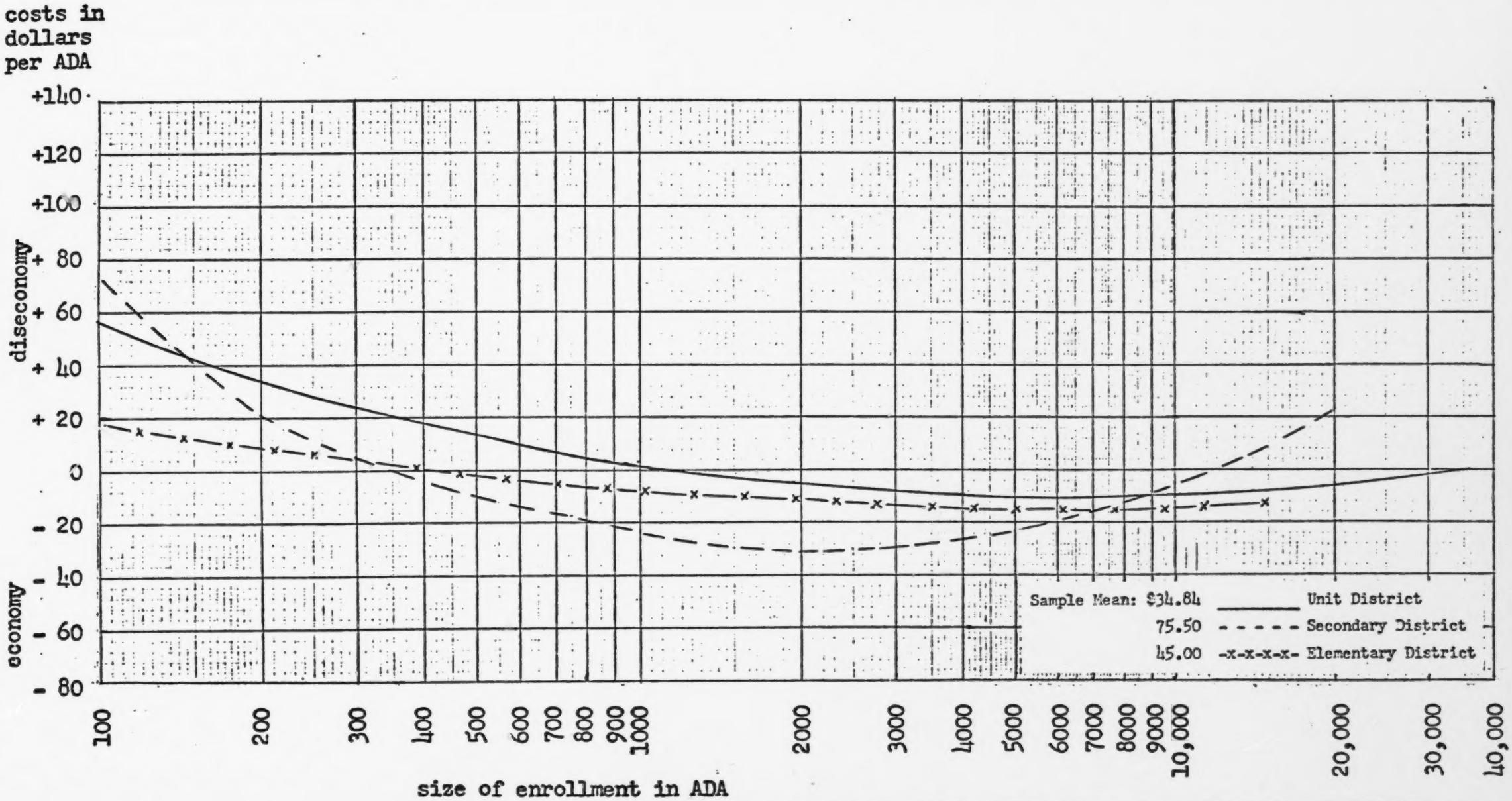


FIGURE 3

ADMINISTRATIVE COST CURVES IN RELATION TO DISTRICT SIZE SHOWING DIFFERENCES FOR THE TYPES OF SCHOOL DISTRICT[a]



[a]Cost expressed in dollars per ADA above or below the mean of the sample.